

Thompson Scattering in an Expanding Universe

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The Thompson cross section for scattering of electromagnetic waves by a free electron in an expanding universe is derived here. The equations of motion of the electron are obtained from the Einstein-Maxwell field equations of general relativity using the Einstein-Infeld-Hoffmann surface integral method. These integrals are evaluated approximately by perturbing off an Einstein-deSitter cosmological field. It is found that the Thompson cross section varies as the inverse square of the cosmic scale factor $R(t)$.

PACS numbers: 03.50.De, 03.80.+r, 04.25.-g, 04.25.Nx

The question of whether or not the constants of nature actually vary on a cosmological time scale was first investigated seriously by Dirac. [1] To satisfy his Large Number hypothesis, Dirac assumed that it was the gravitational constant G that varied on a cosmological time scale. Later [2] he developed this idea further by introducing the notion of gravitational and atomic time and assumed that these two times were not synchronous with each other. [3] However, in a recent work [4] (referred to hereafter as A1), I showed how one could construct model gravitational and electromagnetic clocks whose dynamics in an expanding universe follow directly from the Einstein-Maxwell field equations of general relativity using the Einstein-Infeld-Hoffmann (EIH) [5] surface integral method without other assumptions. It was found that, for a class of clocks for which $T_L \ll T_C \ll T_H$ where T_L is the light travel time across the clock, T_C is the clock period and T_H is the Hubble time, that, to an accuracy of $O(\varepsilon^2\delta)$ where $\varepsilon = T_L/T_C$ and $\delta = T_L/T_H$, the two clocks remain synchronous with each other and measure the same cosmic time t when the cosmic gravitational field $g_{\mu\nu}$ has the form

$$g_{\mu\nu} = \text{diag}(1, -R^2, -R^2, -R^2). \quad (1)$$

In this paper I address a different problem associated with physical processes in an expanding universe, namely Thompson scattering. I will show that the Thompson scattering cross section is in fact not a constant but varies with time by a factor proportional to $1/R^2(t)$.

At first glance it might appear to the reader paradoxical that gravitational effects associated with the expanding universe could have any effect on what is seemingly a purely electrodynamic process. But in fact Thompson scattering is not a purely electromagnetic process. The inertial force term in the equations of motion for the scattering electron is of gravitational origin as EIH first showed and, in the case of an expanding universe, involves the scale factor $R(t)$. As in A1 the exact dependence of the inertial force on $R(t)$ as well as the form of the electromagnetic force due to the incident wave can be determined from the EIH surface integrals by perturbing the gravitational and electromagnetic fields off the Einstein-deSitter field as in A1. However, since one is dealing here with radiation fields it turns out to be more convenient to employ conformal coordinates (τ, x, y, z) in which this field has the form

$$g_{\mu\nu} = R^2(\tau)\text{diag}(1, -1, -1, -1) \quad (2)$$

and where τ and t are related by $dt = R(\tau)d\tau$.

The EIH surface integrals can most easily be derived from the Landau and Lifshitz [6] form of the field equations for $g_{\mu\nu}$ and the electromagnetic field $F_{\mu\nu}$ and have the form (in what follows I will use units in which $G = c = 1$, latin indices run from 1 to 3, greek indices run from 0 to 3, and I employ both the Einstein summation convention and the comma notation to denote partial derivatives)

$$U^{\mu\nu\rho} = \Theta^{\mu\nu} \quad (3)$$

where

$$U^{\mu\nu\rho} = -U^{\mu\rho\nu} = \frac{1}{16\pi} \{ \mathfrak{g}^{\mu\nu} \mathfrak{g}^{\rho\sigma} - \mathfrak{g}^{\mu\rho} \mathfrak{g}^{\nu\sigma} \}_{,\sigma} \quad (4)$$

and

$$\mathfrak{F}^{\mu\nu}_{,\nu} = 0 \quad \text{and} \quad F_{[\mu\nu,\rho]} = 0. \quad (5)$$

In these equations $g = \det(g_{\mu\nu})$, $\mathfrak{g}^{\mu\nu} = \sqrt{-g}g^{\mu\nu}$, $\mathfrak{F}^{\mu\nu} = \sqrt{-g}F^{\mu\nu}$, $\mathfrak{t}_{LL}^{\mu\nu}$ is the Landau-Lifshitz energy-stress pseudotensor and $T^{\mu\nu}$ is the electromagnetic energy-stress tensor given by

$$T^{\mu\nu} = \frac{1}{16\pi} (g^{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} - 4g_{\rho\sigma} F^{\mu\rho} F^{\nu\sigma}). \quad (6)$$

Note that, in our expression for $\Theta^{\mu\nu}$ above, there is no matter contribution from the sources since they are assumed to be compact and to vanish on and outside the EIH surfaces. This feature of the EIH method thereby avoids having to make specific assumptions about the form of the matter energy-stress tensor or the need to introduce singular source terms.

Because of the antisymmetry of $F^{\mu\nu}$ and $U^{\mu\nu\rho}$ in their indices, integration of Eq. (3) over a closed 2-surface in a $t = \text{const.}$ hypersurface gives

$$\oint (U^{\mu r 0},_0 - \Theta^{\mu r}) n_r dS = 0 \quad (7)$$

where n_r is a unit surface normal. In a similar way one gets from Eq. (5) the result

$$\oint \mathfrak{F}^{r 0},_0 n_r dS = 0. \quad (8)$$

It is these last two equations that yield equations of motion for the sources of the gravitational and electromagnetic fields.

When the surfaces over which the integrals in Eqs. (7) and (8) are taken surround a source, the requirement that the surface independent contributions vanish yield these equations. (The surface dependent terms will in all cases vanish either identically or as a consequence of the field equations. [7]) These equations will be used to derive equations of motion for a compact charge in the presence of an incident electromagnetic field whose characteristic length scale is large compared to the size of the EIH surfaces needed to enclose the charge. The near electromagnetic field produced by the motion of this charge is then matched to a far radiation field. The total flux of this scattered field is finally divided by the incident flux to yield an expression for the Thompson scattering cross section.

To evaluate the fields appearing in Eqs. (7) and (8) I follow the methods used in A1 except that here the gravitational field is perturbed off the background field (2) rather than the field (1). For convenience the total field is written as

$$\mathfrak{g}^{\mu\nu} = R^2 \tilde{\mathfrak{g}}^{\mu\nu} \quad (9)$$

and $\tilde{\mathfrak{g}}^{\mu\nu}$ as well as the electromagnetic four-potential A^μ is expanded in a double series in ε and δ . These fields in addition are assumed to depend on the conformal time τ through their dependence on $\varepsilon\tau$ and $\delta\tau$ while R is a function only of δt . (In higher orders of approximation they will of course depend on higher order multiple times.) In addition, charges and masses are scaled so as to be $O(\varepsilon^2)$. In what follows we will need an accuracy of ε^2 and $\varepsilon\delta$ since we are only concerned here with the modifications in the Newtonian dynamics in the background cosmological field.

The lowest order correction to the gravitational field is taken to be [8]

$$\tilde{\mathfrak{g}}^{00} = 1 + \varepsilon^2 h \quad (10)$$

and satisfies

$$\nabla^2 h = 0. \quad (11)$$

For compact spherical sources (and Schwarzschild black holes) the solution in the weak-field zone on and outside the EIH surfaces has the form

$$h = 4 \sum \frac{\tilde{m}_A}{r_A} \quad (12)$$

where the index A labels the sources in the system and the sum is over all A . The \tilde{m}_A are as yet to be determined functions of $\varepsilon\tau$ and $\delta\tau$ and $\mathbf{r}_A = \mathbf{x} - \mathbf{x}_A$ where the \mathbf{x}_A are the coordinates of the A th particle and are also functions of $\varepsilon\tau$ and $\delta\tau$. When the surface integrals in Eq. 7 with $\mu = 0$ are evaluated using this field one finds that

$$\partial_{\varepsilon\tau} \tilde{m}_A = 0 \quad \text{and} \quad R \partial_{\delta\tau} \tilde{m}_A = -\partial_{\delta\tau} R \tilde{m}_A \quad (13)$$

so that, to this order of accuracy,

$$\tilde{m}_A = \frac{m_A}{R} \quad (14)$$

where the m_A are constants. In a like manner one constructs the lowest order contribution to the scalar potential

$$A^0 = \varepsilon^2 \phi \quad (15)$$

with ϕ satisfying

$$\nabla^2 \phi = 0. \quad (16)$$

When the spherically symmetric solution

$$\phi = \sum \frac{\tilde{q}_A}{r_A} \quad (17)$$

is substituted into the surface integral (9) one obtains the result that

$$\partial_{\varepsilon\tau} \tilde{q}_A = \partial_{\delta\tau} \tilde{q}_A = 0 \quad (18)$$

so that

$$\tilde{q}_A = q_A \quad (19)$$

where the q_A are constants.

To derive equations of motion from the surface integrals (7) we need the lowest order corrections to $\tilde{\mathbf{g}}^{0r}$ which are $O(\varepsilon^3)$ and $O(\varepsilon^2\delta)$ so we set

$$\tilde{\mathbf{g}}^{0r} = \varepsilon^3 h_\varepsilon^r + \varepsilon^2 \delta h_\delta^r. \quad (20)$$

There will of course be additional corrections in higher orders of approximation that are small compared to the first correction but large compared to the second, e.g. post-Newtonian corrections of order ε^5 , but here we are only interested in the first order effects of the expanding universe on Newtonian physics. As in A1 h_ε^r and h_δ^r are determined to be

$$h_\varepsilon^r = 4 \sum \frac{\tilde{m}_A}{r_A} x_{A,\varepsilon\tau}^r \quad \text{and} \quad h_\delta^r = 4 \sum \frac{\tilde{m}_A}{r_A} x_{A,\delta\tau}^r. \quad (21)$$

when the gauge conditions

$$h_{\varepsilon,r}^r + \partial_{\varepsilon\tau} h = 0 \quad \text{and} \quad h_{\delta,r}^r + \sum \tilde{m}_A \partial_{\delta\tau} \left(\frac{1}{r_A} \right) = 0 \quad (22)$$

are employed.

Since my purpose here is to examine Thompson scattering in an expanding universe I will, in what follows, confine my attention to a single charged source and take the electromagnetic field used to evaluate $\Theta^{\mu\nu}$ in Eq. (7) to be a plane wave. Maxwell's equations (5) have the solution

$$\mathfrak{F}^{0s} = \delta_3^s E_0 e^{i(\omega\varepsilon\tau - kx)} \quad (23)$$

for a slowly varying wave propagating in the x-direction with $(\omega\varepsilon)^2 - k^2 = 0$. For a wave whose spatial variation is large compared to the size of the EIH surface surrounding the charge the surface integral equation (7) yields the equation of motion

$$m_1 x_{1,\varepsilon\tau\varepsilon\tau}^r = -\frac{1}{R} q E_0 e^{i\omega\varepsilon\tau} \quad (24)$$

which has the solution

$$x_1^3 = \frac{1}{R} \frac{q_1}{m_1} \frac{1}{\omega^2} E_0 e^{i\omega\varepsilon\tau}. \quad (25)$$

It remains finally to compute the scattered field produced by the motion of our charge. One possible way to do this would be to introduce a model source term into the right hand side of the first of Eqs. (5) whose motion is characterized by x_1^3 above. However, in keeping with the EIH philosophy of not specifically introducing sources into the field equations and because the EIH procedure makes it unnecessary, we will proceed by constructing a radiation

solution whose inner expansion matches on to the outer expansion of the near field ϕ in Eq. (17) with r_A computed using x_1^3 above. This outer expansion in inverse powers of r is given by

$$\begin{aligned}\phi &= \frac{q_1}{(r^2 + r_1^2 - 2rr_1 \cos \theta)^{1/2}} \\ &= \frac{q_1}{r} + \frac{q_1}{r^2} r_1 \cos \theta + O(1/r^3).\end{aligned}\tag{26}$$

The corresponding outer dipole field is given by

$$\phi = \frac{a}{\varepsilon r} + \left\{ \frac{W'(\varepsilon(\tau - r))}{\varepsilon r} + \frac{W(\varepsilon(\tau - r))}{(\varepsilon r)^2} \right\} \cos \theta\tag{27}$$

whose inner expansion is

$$\phi = \frac{a}{\varepsilon r} + \frac{W(\varepsilon\tau)}{(\varepsilon r)^2} + O(1)\tag{28}$$

where a and $W(\varepsilon\tau)$ are to be determined by matching. Comparing Eqs. (26) and (28) we see that

$$a = \varepsilon q_1 \quad \text{and} \quad W(\varepsilon\tau) = \varepsilon^2 q_1 r_1.\tag{29}$$

The gauge condition $A^\mu{}_{,\mu} = 0$ used to derive the wave equations for A^μ allows us to derive an expression for the vector potential part of A^μ corresponding to ϕ given by

$$A_3 = -\frac{\varepsilon q_1 r_1 \varepsilon \tau}{r}.\tag{30}$$

With the above expressions for ϕ and A_3 we can now calculate $F_{\mu\nu}$ which in turn allows us to determine the Poynting vector $S^r = T^{0r}$ to be

$$S^r = \frac{1}{4\pi} \frac{q_1^2}{r^2} \{ (1 + n_3)n_1, (1 + n_3)n_2, -(n_1^2 + n_2^2) \} (r_{1,\tau\tau})^2 + O\left(\frac{1}{r^3}\right).\tag{31}$$

where n_r is a unit vector. The total integrated flux f is then calculated to be

$$f = \frac{2}{3} q_1^2 (r_{1,\tau\tau})^2 = \frac{2}{3} \left(\frac{1}{R} \frac{q_1^2}{m_1} E_0 \right)^2.\tag{32}$$

The Thompson scattering cross section σ_T , equal to f divided by the incoming flux $E_0^2/4\pi$, is thus given by

$$\sigma_T = \frac{8\pi}{3} \left(\frac{1}{R} \frac{q_1^2}{m_1} \right)^2\tag{33}$$

and is seen to differ from the usual expression for σ_T by the factor of $1/R^2$.

This result raises a number of complex questions. First and foremost, is it true or does some mechanism shield elementary processes such as Thompson scattering from the effects of the expanding universe? Do all cross sections, e.g. nuclear, experience this same effect? If the answer to both questions is yes, then a number of issues in stellar evolution and nucleosynthesis in the early universe will of necessity have to be reexamined.

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